BARTLETT

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Unit 2, Lesson 4: Proportional Relationships and Equations

Let's write equations describing proportional relationships.

4.1: Number Talk: Division

Display one at a time
Find each quotient mentally.
1 min think/work (independent), then pair, then class discussion

2-3 strategies per problem
ask for reasoning

1) $645 \div 100$
= 6.45

2) $645 \div 50$
= 12.9

3) $48.6 \div 30$ = 1.62

pause - we will use patterns from above to answer last

4) $48.6 \div x$

e.g.
x = 60, then = 0.81
x = 15, then = 3.24

choose a value for x in which you could easily find the quotient.

use what you know about the equation to reason about the last expression

4.2: Feeding a Crowd, Revisited

5 mins work, partners

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

$\frac{y}{x} = k$

x ·3 y

cups of dry rice	number of people
1	3
2	6
3	9
12	36
43	129
x	$3x$

$\cdot \frac{1}{3}$

a. How many people will 1 cup of rice serve? One cup of rice will serve three people.

b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

3 cups of rice will serve 9 people, 12 cups will serve 36, and 43 cups will serve 129.

c. How many people will x cups of rice serve?

x cups of rice will serve 3x people.

k = 3

The constant of proportionality is 3.

$2 \cdot \boxed{?} = 6$
$\div 2 \quad \div 2$
$1 \cdot \boxed{?} = 3$

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2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 1 spring roll serve?

1 spring roll will feed 1/2 a person

b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?

10 spring rolls will feed 5 people, 16 spring rolls will feed 8, and 25 spring rolls will feed 12 1/2.

c. How many people will n spring rolls serve?

n spring rolls will feed $\frac{1}{2}n$ people.

number of spring rolls	number of people
1	1/2
6	3
10	5
16	8
25	12.5
n	$\frac{1}{2}n$

•1/2

•2

If difficult:
- draw diagrams
- verbalize the relationship in words

3. How was completing this table different from the previous table? How was it the same?

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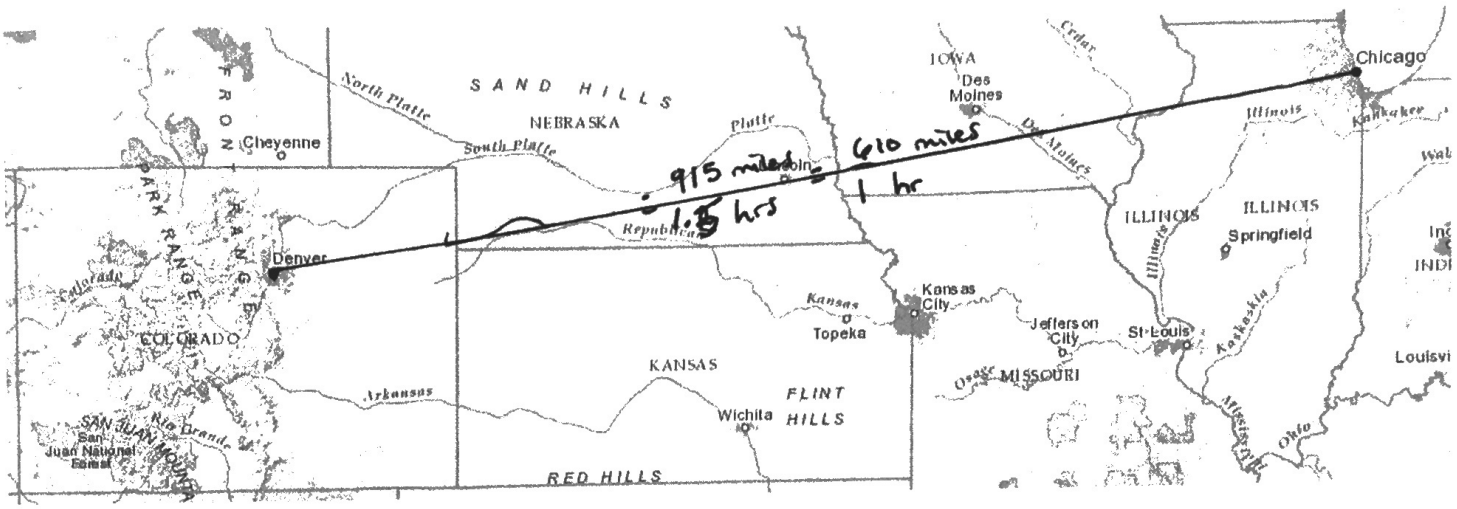
5 min work, partners

Speed = $\frac{\text{distance traveled}}{\text{time spent traveling}}$

4.3: Denver to Chicago

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.



1. Complete the table.

x y k ← constant of proportionality

time (hours)	distance (miles)	speed (miles per hour)
1	610	610
1.5	915	610
2	1,220	610
2.5	1,525	610
t	$610t$	610

$\frac{915}{1.5} = k$

2. How far does the plane fly in one hour?

The plane flies 610 miles in 1 hour.

3. How far would the plane fly in t hours at this speed?

The plane would fly $610t$ miles at this speed.

4. If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d.

$d = 610t$ or $\frac{d}{t} = 610$ or $\frac{d}{610} = t$

5. How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

The plane would fly 1,830 miles in 3 hours at this speed; in 3.5 hours it would fly 2,135 miles.

610 × 3 = 1830

610 × 3.5 = 2135

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Are you ready for more?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach the Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)

   $1.3 \cdot 3.26 \approx 4.24$ or about 4.24 years

2. There are two twins. One twin leaves on a spaceship to explore the planet near Proxima Centauri traveling at 90% of the speed of light, while the other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri? (Do you think the answer would be the same for the other twin? Consider researching "The Twin Paradox" to learn more.)

   $\frac{4.24}{0.9} \approx 4.7$ or 4.7 years

4.4: Revisiting Bread Dough 5 min work, partners

A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of honey to flour.

1. Complete the table.

honey (tbsp)	flour (c)
1	$\frac{5}{4}$
8	10
16	20
30	24
h	$\frac{5}{4}h$

2. If f is the cups of flour needed for h tablespoons of honey, write an equation that relates f and h.

   $f = \frac{5}{4}h$ or $\frac{4}{5}f = h$

3. How much flour is needed for 15 tablespoons of honey? 17 tablespoons? Explain or show your reasoning.

   12 cups of flour is needed for 15 tbspns of honey,
   $13\frac{3}{5}$ (13.6) cups for 17 tbspns.

   $\frac{15}{1} \cdot \frac{5}{4} = \frac{}{4}$

   $\frac{17}{1} \cdot \frac{5}{4} = \frac{}{4}$

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Unit 2, Lesson 5: Two Equations for Each Relationship

Let's investigate the equations that represent proportional relationships.

5.1: Missing Figures

Here are the second and fourth figures in a pattern.

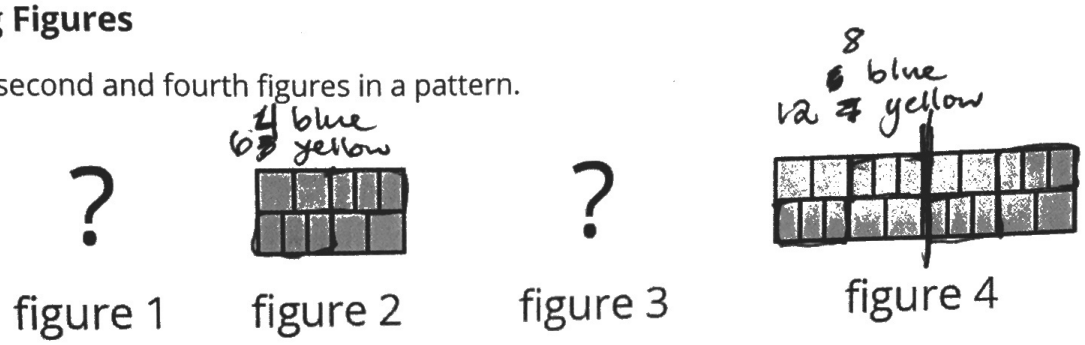

figure 1 figure 2 figure 3 figure 4

1. What do you think the first and third figures in the pattern look like?

2. Describe the 10th figure in the pattern.

Figure 10 would have 18 20 blue & 30 yellow tiles.

NAME Q: Where does the constant of proportionality occur in each table? DATE PERIOD

5.2: Meters and Centimeters

5 mins work, then partner

100 k = 1

There are 100 centimeters (cm) in every meter (m).

Table 1

· 100

length (m)	length (cm)
1	100
0.94	94
1.67	167
57.24	5,724
x	$100x$

· 1/100 or 0.01

Table 2

· 1/100 or 0.01

length (cm)	length (m)
100	1
250	2.5
78.2	0.782
123.9	1.239
y	$0.01y$ or $\frac{1}{100}y$

· 100

1. Complete each of the tables.

2. For each table, find the constant of proportionality.

The constant of Table 1 is 100, the constant of Table 2 is 0.01.

3. What is the relationship between these constants of proportionality?

The relationship between the constants is reciprocal.

4. For each table, write an equation for the proportional relationship. Let x represent a length measured in meters and y represent the same length measured in centimeters.

Table 1

$y = 100x$

Table 2

$x = 0.01y$ or $x = \frac{1}{100}y$ or $x = \frac{y}{100}$

Are you ready for more?

1. How many cubic centimeters are there in a cubic meter?

1,000,000 cubic cm are in a cubic m

2. How do you convert cubic centimeters to cubic meters?

Multiply by $\frac{1}{1,000,000}$

3. How do you convert the other way?

Multiply by 1,000,000

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5.3: Filling a Water Cooler 5 mins work, then partner

It took Priya 5 minutes to fill a cooler with 8 gallons of water from a faucet that was flowing at a steady rate. Let w be the number of gallons of water in the cooler after t minutes.

1. Which of the following equations represent the relationship between w and t? Select **all** that apply.

A. $w = 1.6t$
B. $w = 0.625t$
C. $t = 1.6w$
D. $t = 0.625w$

2. What does 1.6 tell you about the situation?

1.6 tells you that water is flowing at 1.6 gallons per minute.

3. What does 0.625 tell you about the situation?

0.625 tells you that ~~water~~ it takes 0.625 mins for 1 gallon of water to flow.

4. Priya changed the rate at which water flowed through the faucet. Write an equation that represents the relationship of w and t when it takes 3 minutes to fill the cooler with 1 gallon of water.

$t = 3w$ or $w = \frac{1}{3}t$

5. Was the cooler filling faster before or after Priya changed the rate of water flow? Explain how you know.

The cooler filled faster before Priya changed the rate of water flow.

Before it took 0.625 mins to get one gallon, but after, it took 3 mins to get one gallon.

Before = 1.6 gallons / min

After = 1/3 gallons / min

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5.4: Feeding Shrimp (optional)

At an aquarium, a shrimp is fed $\frac{1}{5}$ gram of food each feeding and is fed 3 times each day.

1. How much food does a shrimp get fed in one day?

   $\frac{3}{5}$ grams

2. Complete the table to show how many grams of food the shrimp is fed over different numbers of days.

   $\cdot \frac{3}{5}$

number of days	food in grams
1	$\frac{3}{5}$ or 0.6
7	$4\frac{1}{5}$ or 4.2
30	18

3. What is the constant of proportionality? What does it tell us about the situation?

   $\frac{3}{5}$

4. If we switched the columns in the table, what would be the constant of proportionality? Explain your reasoning.

   $\frac{5}{3}$ - reciprocal

5. Use d for number of days and f for amount of food in grams that a shrimp eats to write *two* equations that represent the relationship between d and f.

   $f = \frac{3}{5}d$ and $d = \frac{5}{3}f$

6. If a tank has 10 shrimp in it, how much food is added to the tank each day?

   6 grams are added each day.

7. If the aquarium manager has 300 grams of shrimp food for this tank of 10 shrimp, how many days will it last? Explain or show your reasoning.

   It will last 50 days.

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Lesson 4 Summary

The table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

Note that "parts" can be *any* unit for volume. If we mix 3 cups of red with 12 cups of blue, you will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

The last row in the table says that if we know the amount of red paint needed, r, we can always multiply it by 4 to find the amount of blue paint needed, b, to mix with it to make Venusian Sunset. We can say this more succinctly with the equation $b = 4r$. So the amount of blue paint is proportional to the amount of red paint and the constant of proportionality is 4.

We can also look at this relationship the other way around.

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

If we know the amount of blue paint needed, b, we can always multiply it by $\frac{1}{4}$ to find the amount of red paint needed, r, to mix with it to make Venusian Sunset. So $r = \frac{1}{4}b$. The amount of blue paint is proportional to the amount of red paint and the constant of proportionality $\frac{1}{4}$.

In general, when y is proportional to x, we can always multiply x by the same number k—the constant of proportionality—to get y. We can write this much more succinctly with the equation $y = kx$.

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Lesson 5 Summary

If Kiran rode his bike at a constant 10 miles per hour, his distance in miles, d, is proportional to the number of hours, t, that he rode. We can write the equation

$$d = 10t$$

With this equation, it is easy to find the distance Kiran rode when we know how long it took because we can just multiply the time by 10.

We can rewrite the equation:

$$\begin{aligned} d &= 10t \\ \left(\frac{1}{10}\right) d &= t \\ t &= \left(\frac{1}{10}\right) d \end{aligned}$$

This version of the equation tells us that the amount of time he rode is proportional to the distance he traveled, and the constant of proportionality is $\frac{1}{10}$. That form is easier to use when we know his distance and want to find how long it took because we can just multiply the distance by $\frac{1}{10}$.

When two quantities x and y are in a proportional relationship, we can write the equation

$$y = kx$$

and say, "y is proportional to x." In this case, the number k is the corresponding constant of proportionality. We can also write the equation

$$x = \frac{1}{k}y$$

and say, "x is proportional to y." In this case, the number $\frac{1}{k}$ is the corresponding constant of proportionality. Each one can be useful depending on the information we have and the quantity we are trying to figure out.