BARTLETT

NAME DATE PERIOD

$y = kx$

$\frac{y}{x} = k$

$x = \frac{y}{k}$

Unit 2, Lesson 6: Using Equations to Solve Problems

Let's use equations to solve problems involving proportional relationships.

6.1: Number Talk: Quotients with Decimal Points

1. Without calculating, order the quotients of these expressions from least to greatest.

Display #1 first

2 mins work, whole class discuss

4) $42.6 \div 0.07$ greatest $= 608.571$

1) $42.6 \div 70$ least $= 0.608571$

3) $42.6 \div 0.7$ $= 60.8571$

2) $426 \div 70$ $= 6.08571$

Display #2 - 1 min work, pair-share, whole class

$426 \div 70 = 6.08571$

2. a. Place the decimal point in the appropriate location in the quotient: $42.6 \div 7 = 608571$

6.08571

b. Use this answer to find the quotient of *one* of the previous expressions.

6.2: Concert Ticket Sales

A performer expects to sell 5,000 tickets for an upcoming concert. They want to make a total of \$311,000 in sales from these tickets.

$y = 62.2x$

1. Assuming that all tickets have the same price, what is the price for one ticket?

\$62.20 for every ticket sold.

$$\begin{array}{r} 62.20 \\ 5000 \overline{)\,311000.0} \\ -30000 \\ \hline 11000 \\ -10000 \\ \hline 10000 \\ -10000 \\ \hline 0 \end{array}$$

2. How much will they make if they sell 7,000 tickets?

$$\begin{array}{r} 7000 \\ \times 62.2 \\ \hline 435{,}400 \end{array}$$

\$435,400 are made for 7,000 tickets sold

3. How much will they make if they sell 10,000 tickets? 50,000? 120,000? a million? x tickets?

They will make: \$622,000 \$3,110,000 \$7,464,000 \$62,200,000 $62.2x$

for each value.

4. If they make \$379,420, how many tickets have they sold?

6,100 tickets are sold for \$379,420

5. How many tickets will they have to sell to make \$5,000,000?

80,386 tickets must be sold to make \$5,000,000

6.3: Recycling

5 mins work, pair-share, whole class

Aluminum cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminum cans is 0.16 kilograms. The aluminum in 10 cans that are recycled has a value of \$0.14.

1. If a family threw away 2.4 kg of aluminum in a month, how many cans did they throw away? Explain or show your reasoning.

   150 cans because 2.4 is $(0.16) \cdot 15$, and

   $10 \cdot 15 = 150$

2. What would be the recycled value of those same cans? Explain or show your reasoning.

   \$2.10, because $(0.14) \cdot 15 = 2.1$

3. Write an equation to represent the number of cans c given their weight w.

   ~~[illegible]~~

   $c = 62.5w$

4. Write an equation to represent the recycled value r of c cans.

   ~~[illegible]~~ $r = 0.014c$

5. Write an equation to represent the recycled value r of w kilograms of aluminum.

   $r = 0.875w$ or $r = 7/8\,w$

Are you ready for more?

The EPA estimated that in 2013, the average amount of garbage produced in the United States was 4.4 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2,000 pounds.)

≈ 32 ½ weeks (family of 2) ≈ 21 ½ weeks (family of 3) ≈ 15 ½ weeks (family of 4)

Lesson 6 Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

equation that proves proportionality

For example, we know that Denali, the highest mountain peak in North America, is 20,300 feet above sea level. How many miles is that? There are 5,280 feet in 1 mile. This relationship can be represented by the equation

$$f = 5{,}280m$$

where f represents a distance measured in feet and m represents the same distance measured miles. Since we know Denali is 20,310 feet above sea level, we can write

$$20{,}310 = 5{,}280m$$

So $m = \frac{20{,}310}{5{,}280}$, which is approximately 3.85 miles.

NAME DATE PERIOD

Unit 2, Lesson 7: Comparing Relationships with Tables

Let's explore how proportional relationships are different from other relationships.

7.1: Adjusting a Recipe 2 mins think,

A lemonade recipe calls for the juice of 5 lemons, 2 cups of water, and 2 tablespoons of honey.

Invent four new versions of this lemonade recipe:

Answers vary; sample responses:

1. One that would make more lemonade but taste the same as the original recipe.

   10 lemons, 4 cups of water, 4 tbspns honey

2. One that would make less lemonade but taste the same as the original recipe.

   $2\frac{1}{2}$ lemons, 1 cup of water, 1 tbspn of honey

3. One that would have a stronger lemon taste than the original recipe.

   8 lemons, 2 cups water, 2 tbspns honey

4. One that would have a weaker lemon taste than the original recipe.

   2 lemons, 2 cups water, 2 tbspns honey

- Take note of patterns that emerge among responses
- @ least 3 responses for each
- Q: How did the ratios change in the new recipes?

NAME DATE PERIOD

5 mins quiet work time, pair-share, whole class

7.2: Visiting the State Park (Not a proportional relationship)

Entrance to a state park costs $6 per vehicle, plus $2 per person in the vehicle. $y = 2x + 6$

1. How much would it cost for a car with 2 people to enter the park? 4 people? 10 people? Record your answers in the table.

number of people in vehicle (x)	total entrance cost in dollars (y)	
2	$10	$2 \cdot 2 + 6$
4	$14	$2 \cdot 4 + 6$
10	$26	$2 \cdot 10 + 6$

(unit rates)

2. For each row in the table, if each person in the vehicle splits the entrance cost equally, how much will each person pay?

$5.00 (2 people)
$3.50 (4 people)
$2.60 (10 people)

can't just scale up from 10 (due to $6 charge per vehicle)

3. How might you determine the entrance cost for a bus with 50 people?

$\$106 = 2 \cdot 50 + 6$

4. Is the relationship between the number of people and the total entrance cost a proportional relationship? Explain how you know.

- No, not equivalent ratios between x and y.
- No, cost per person changes depending on the number of people.
- No, each set of values is not characterized by the same unit rate.

Are you ready for more?

What equation could you use to find the total entrance cost for a vehicle with any number of people?

p = number of people c = total cost in $

$c = 6 + 2p$ or $c = 2p + 6$

NAME DATE PERIOD

7.3: Running Laps

5 mins work, 5 mins pair share, whole class

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps) d	time (minutes) t	minutes per lap pace
2	4	2
4	9	2.25 or $2\frac{1}{4}$
6	15	2.5 or $2\frac{1}{2}$
8	23	2.875 or $2\frac{7}{8}$

Q: Can you represent the table with an equation?

Clare's run:

distance (laps) d	time (minutes) t	minutes per lap pace
2	5	2.5 or $2\frac{1}{2}$
4	10	2.5
6	15	2.5
8	20	2.5

Q: What if we calculate laps / min vs. mins / lap?

($d = 0.4t$ for Clare)

1. Is Han running at a constant pace? Is Clare? How do you know?

Han is not because his pace changes.

Clare might be. Her pace is constant, but she might not be running at a constant pace (e.g. she might stand still for half a minute, then complete a lap in 2 mins)

2. Write an equation for the relationship between distance and time for anyone who is running at a constant pace.

$t = 2.5d$ for Clare's table.

t = time
d = distance

NAME DATE PERIOD

Lesson 7 Summary

Here are the prices for some smoothies at two different smoothie shops:

Smoothie Shop A

smoothie size (oz)	price ($)	dollars per ounce
8	6	0.75
12	9	0.75
16	12	0.75
s	$0.75s$	0.75

Smoothie Shop B

smoothie size (oz)	price ($)	dollars per ounce
8	6	0.75
12	8	0.67
16	10	0.625
s	???	???

For Smoothie Shop A, smoothies cost $0.75 per ounce no matter which size we buy. There could be a proportional relationship between smoothie size and the price of the smoothie. An equation representing this relationship is

$$p = 0.75s$$

If proportional, this equation would work

where s represents size in ounces and p represents price in dollars. (The relationship could still not be proportional, if there were a different size on the menu that did not have the same price per ounce.) || * not necessarily proportional

For Smoothie Shop B, the cost per ounce is different for each size. Here the relationship between smoothie size and price is definitely *not* proportional.

In general, two quantities in a proportional relationship will always have the same quotient. When we see some values for two related quantities in a table and we get the same quotient when we divide them, that means they might be in a proportional relationship—but if we can't see all of the possible pairs, we can't be completely sure. However, if we know the relationship can be represented by an equation is of the form $y = kx$, then we are sure it is proportional.

Lesson Take-Aways:
- If the quotient is the same for each row, the table could be proportional
- The quotient IS the constant of proportionality (if proportional)
- If not all quotients are the same, NOT proportional
- In a proportional relationship, can express using the equation $y = kx$